Analytical 3D Approach to Simultaneous Compensation for Photon Attenuation and Collimator Response in Quantitative Fan-Beam Collimated Brain SPECT

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Abstract – **Fan-beam collimation provides the optimal geometry for data collection for brain SPECT imaging. For a non-parallel projection geometry, there is no symmetry on the projection rays, except the periodical property of the projection angle. This property is well defined by circular harmonic decomposition (CHD). This paper utilizes the CHD to explore the fan-beam collimation geometry, including photon attenuation and collimator response, for quantitative brain SPECT in three dimensions. An analytical solution compensating simultaneously for both of the photon attenuation and collimator responses is presented. An efficient algorithm for the solution is formulated and implemented by fast Fourier transforms. The reconstruction has been validated by experiments on the Shepp-Logan and Hoffman brain phantoms at various noise levels.**

Index terms**—circular harmonic decomposition, point spread function, fan-beam collimation.**

I. Introduction

Single photon emission computed tomography (SPECT) can provide quantitative information of the tissue functionality in three-dimensions (3D). However, two major problems that are causing difficulties in image reconstruction for quantitative SPECT arise from the absorption of γ-rays by the body and the non-stationary point spread response of the collimator (the scatter of the γ-rays can be treated by other means and will not be discussed here). For brain SPECT, the attenuation problem can be simplified, because there are negligible radioactive nuclides distributed inside the skull and scalp and furthermore the attenuation of the skull and scalp can be equivalent to that of an enlarged brain tissue of a constant attenuation coefficient, as a compensation for uniform attenuation in a convex region[1]. Collimator blurring makes the image reconstruction more complicated, even for the uniform attenuation and parallel-hole collimators[2]. There does not exist a complete analytical algorithm by now that compensates accurately both the attenuation and the point spread response effects simultaneously, especially for the fan beam collimator geometry in 3D. L. van Elmbt and S. Walrand considered the problem for parallel geometry with approximated algorithm[3], while E. J. Soares et al attacked the problem of the same geometry for some particular resolution variation functions, such as the Cauchy model[4]. Other researchers correct either for the non-stationary resolution variation or for the constant attenuation, but not both. And most of these work are either for 2D applications or for parallel hole collimators[5-9].

Although many iterative reconstruction algorithms can do the job and are flexible to be applied for many kinds of complicated collimator geometry[10-12], the computing burden is always the drawback. If an analytical inversion formula can be derived for the solution of the projection equation, it is an interesting research topic for further investigation for practical use. The derivation itself is also an interesting research topic.

Fan-beam collimation is an optimal geometry for brain SPECT. It offers no symmetry for the projections, except the periodical property of projection angle. Circular harmonic decomposition (CHD) has been widely used to explore the property. In this paper, we present an analytical inversion solution that simultaneously compensates for both photon attenuation and collimator response of the fan-beam collimated SPECT system. This method considers the collimators blurring effect and intrinsic detector response together as a system point spread function (PSF). It doesn't limit the PSF in certain forms, but only based on the assumption that the PSF is valid to each individual collimator hole. Our method can be applied to parallel-hole, fan-beam, or varying focal-length fan-beam collimator geometry. It greatly reduced the burden of computational complexity. It was validated under a more realistic case of a Gaussian response function whose FWHM (full width at half maximum) is a function of relative distances (lateral and normal to the collimator surface) on both the Shepp-Logan ellipse phantom and Hoffman brain phantom.

II. Theory

A typical fan-beam collimator geometry is shown by Figure 1, restricted on a single slice. The object activity distribution $f(x,y,z)$ is of our interest.

With the inclusion of uniform attenuation of the brain tissues and an 3D collimator response, the measured projection data $p_m(s,z,\theta)$ is expressed as:

$$p_m(s,\theta,z) = \iiint f(x',y',z') e^{-\mu d(s,\theta,z;x',y',z')} k(s,\theta,z;x',y',z') dx'dy'dz' \quad (1)$$

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where $k(.)$ is the system point spread function. Let the lateral distance to each individual collimator hole being labeled as s, see Figure 1, and a Cartesian coordinate system of l-t is chosen such that the t axis is parallel to the focusing direction of this hole, then the system response function $k(.)$ can be assumed as:

$$k = k\left(l' - l(s), z' - z, t' + R\right). \qquad (2)$$

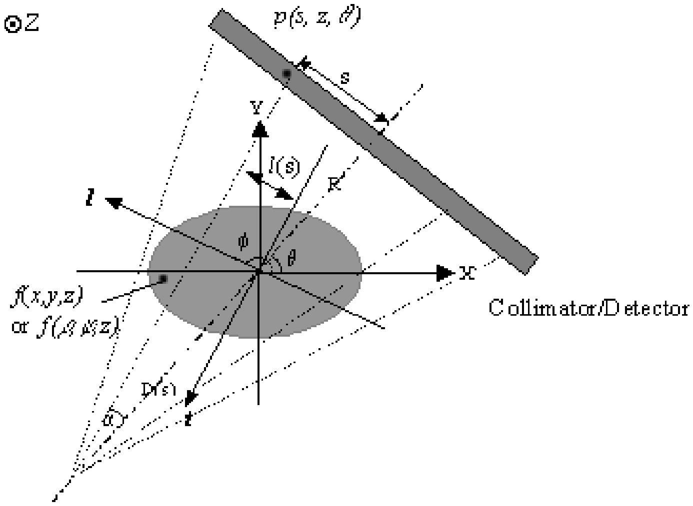

Figure 1. Fan-beam geometry and the coordinate systems**.**

This assumption simply means that the system response is a function of the distances from source activity point to the collimator hole in 3D. This PSF assumption has been investigated by many researchers and is consistent in those references [13-18].

Following the method as described by Bellini et al [19] in dealing with the constant attenuation, we define $L(s,\theta)$ as the distance from point (s,θ) on detector to the boundary of the object (s,t) with t>0, and let $p_\mu(s,z,\theta) = exp[-\mu L(s,\theta)+\mu R]\ p(s,z,\theta)$, then (1) become:

$$\begin{aligned} &p_\mu(s,\theta,z) \\ &= \iiint f(x',y',z') e^{-\mu t'} k(l(s)-l', z-z', t'+R)\, dx' dy' dz' \end{aligned} \qquad (3)$$

By polar coordinate system, (2) can be rewritten as:

$$\begin{aligned} &p_\mu(s,\theta,z) \\ &= \int_0^R \rho d\rho \int_0^{2\pi} d\varphi \int_{-\infty}^{\infty} dz' f'(\rho,\varphi,z') k\left(\rho\sin(\varphi-\theta-\alpha) - l(s), z'-z, \rho\cos(\varphi-\theta-\alpha)\right) \\ &= \int_0^R d\rho \int_0^{2\pi} d\varphi \int_{-\infty}^{\infty} dz' f'(\rho,\varphi,z') k'\left(\rho, l(s); (\varphi-\theta), z'-z\right) \end{aligned} \qquad (4)$$

where the relations held as follows:

$$\phi = \theta + \alpha + \pi/2\ , \qquad l(s) = \frac{D\cdot s}{\sqrt{(D+R)^2 + s^2}},$$

$$\alpha = \alpha(s) = \tan^{-1}\left(\frac{R+D}{s}\right)\ , \qquad \begin{pmatrix} l \\ t \end{pmatrix} = \begin{pmatrix} \rho\sin(\varphi-\theta-\alpha) \\ -\rho\cos(\varphi-\theta-\alpha) \end{pmatrix} \qquad (5)$$

In general we may not have a closed inversion form for formula (4). The purpose of this paper is to derive an optimal estimation of $f(x,y,z)$ or $f(\rho,\varphi,z)$ based on the equation. In the following, we will use the CHD technique to solve this simultaneous compensation problem. The theoretical procedure is given as follows.

By Fourier transform on (4) with respect to variable z, we have:

$$\tilde{p}(s,\theta,\xi) = \int_0^R d\rho \int_0^{2\pi} d\varphi\, \tilde{f}(\rho,\varphi,\xi) \tilde{k}\left(l(s), \rho; \theta-\varphi, \xi\right) \qquad (6)$$

where $\tilde{k}$ denotes the Fourier transform of k' for the variable z, and $\tilde{p}$ and $\tilde{f}$ have the similar meanings. Note that the image property $\tilde{f}(\rho,\varphi,\xi)$ can be estimated slice-by-slice for ξ, indicating that the compensation for the 3D collimator response may be uncoupled to a 2D compensation. This is because that the PSF is shift invariant along the rotation z-axis. By computing the Fourier series expansion of θ on both sides of (6) (this is usually called circular harmonic decomposition of a function in mathematics), we have the following formula:

$$\hat{p}(s,n,\xi) = \int_0^R d\rho\ \hat{f}^*(\rho,n,\xi) \hat{k}\left(l(s), \rho; n, \xi\right) \qquad (7)$$

where

$$\hat{p}(s,n,\xi) = \frac{1}{2\pi} \int_0^{2\pi} d\theta\ \tilde{p}(\rho,\theta,\xi) e^{-in\theta}$$

$$\hat{f}(s,n,\xi) = \frac{1}{2\pi} \int_0^{2\pi} d\varphi\ \tilde{f}(\rho,\varphi,\xi) e^{-in\varphi}, \qquad (8)$$

and notation * stands for conjugate operation. Obviously, (7) is only a CHD expression of (6). Now the reconstruction problem becomes a task of solving a linear algebraic equation for each n and ξ :

$$\begin{pmatrix} \tilde{p}(s_0) \\ \tilde{p}(s_1) \\ \tilde{p}(s_2) \\ \ldots\ldots \end{pmatrix} = \begin{pmatrix} M(s_0\rho_0) & M(s_0\rho_1) & M(s_0\rho_2) & \ldots\ldots \\ M(s_1\rho_0) & M(s_1\rho_1) & M(s_1\rho_2) & \ldots\ldots \\ M(s_2\rho_0) & M(s_2\rho_1) & M(s_2\rho_2) & \ldots\ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots \end{pmatrix} \begin{pmatrix} \tilde{f}(\rho_0) \\ \tilde{f}(\rho_1) \\ \tilde{f}(\rho_2) \\ \ldots\ldots \end{pmatrix}$$

$$\boldsymbol{P=Mf} \qquad (8)$$

Given p and M, and looking for f, this is a typical linear problem that could be solved by several methods. Here we are using the conjugate gradient algorithm. Since the dimension is significantly reduced from 3D to 2D, efficient calculation is expected.

III. IMPLEMENTATION

Fan-beam projections of the Shepp-Logan ellipse phantom and Hoffman brain phantom with different $^{99}Tc^m$ activity concentrations were simulated on a circular orbit of 64 evenly spaced views, taking into account the photon attenuation and collimator response effects. Both the noise-free projection data and their noisy versions with Poisson noise were used to test the reconstruction algorithm.

The reconstruction procedure is as follows:

1. Perform the transforms on PSF matrix *M*. This task is a pre-calculated step given the PSF, before image reconstruction.
2. Perform Fourier transform on the projection data for variable *z*, and Fourier series expansion for variable θ. Note that since the PSF is periodic about θ, so zero padding is not necessary. But for variable *z* (the shift invariant characteristics), zero padding may be needed.
3. Utilize the conjugate gradient algorithm to solve the linear equation (8).
4. After the $\tilde{f}(\rho, n, \xi)$ is obtained, perform two times of Fourier inversion transform, relating to *z* and θ, to find the source image.

In our experiment, the response function was chosen as the most general Gaussian function. Because of the CHD method, transforms between Cartesian coordinate system and polar coordinate system is needed.

IV. RESULTS AND DISCUSSION

The simulation results of different reconstruction procedures are shown in Figure 2, which gives four arbitrary slices for illustration and comparison purposes. Similar results were obtained for the Shepp-Logan phantom. The first row contains slices from the original Hoffman brain phantom. The second row shows the reconstructed results from the blurred projection data containing the photon attenuation and collimator response effects. The third row represents the reconstructed images from projection data with Poisson noise at a noise level similar to a typical clinic study. For a 64x64x32 image, the reconstruction time was less than one minute on a PC platform with Pentium III 550 processor.

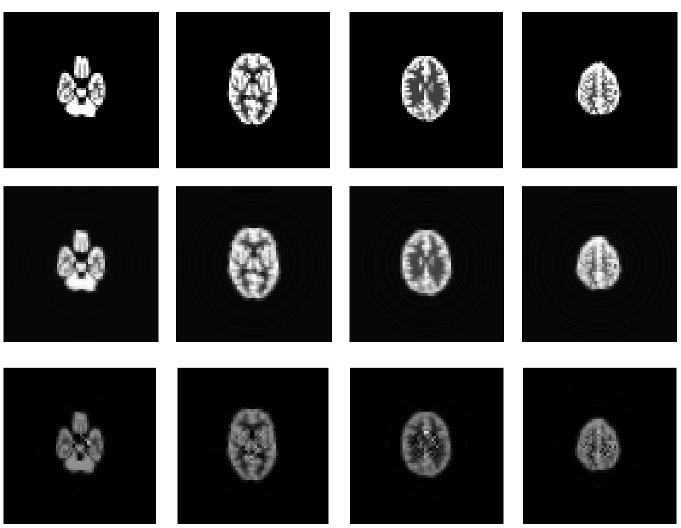

Figure 2. Hoffman brain phantom results. Top row represents the original Hoffman phantom slices. Second row is the reconstructed image from noise-free projection data with attenuation and PSF effects included. Bottom row is the reconstructed image from projection data containing Poisson noises.

V. CONCLUSION

First, our method inherits the idea about angular expression in our previous work[20,21], which is beneficial for non-parallel ray collimator geometry. It is not a back-projection method of source and projections with compensation for photon attenuation and resolution variation. It constructs a new relationship between source image and projection data using CHD technique. In this relationship, the photon attenuation and collimator response can be simultaneously considered in 3D. The compensation can be implemented slice-by-slice along the rotation direction after Fourier transform, because of the shift invariance along that direction. Second, estimation of source image is efficient and accurate via the conjugate gradient algorithm, which converges in finite iterations, as demonstrated by Figure 2. The simulation study revealed this optimization of the proposed algorithm. The order of matrix *M* in (8) is only *N*x*N* size, and the calculation for *N* slices can be performed in parallel at the same time, so the computation is very efficient.

This method considers the periodical property of projection rays of non-parallel-hole collimator geometry and the shift invariant characteristics of fan-beam configuration along the rotation direction. The periodic property is explored by the CHD technique. The shift invariant characteristics are efficiently utilized in the Fourier space. This strategy reduces the 3D PSF treatment into 2D task in the Fourier space and, therefore, improves the computing efficiency. For the reduced matrix size, the conjugate gradient method is a choice for the calculation. The computer simulation is encouraging. Further validation by physical phantom experiments is under progress.

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